Solution of Assignment # 3

Moment & Equilibrium

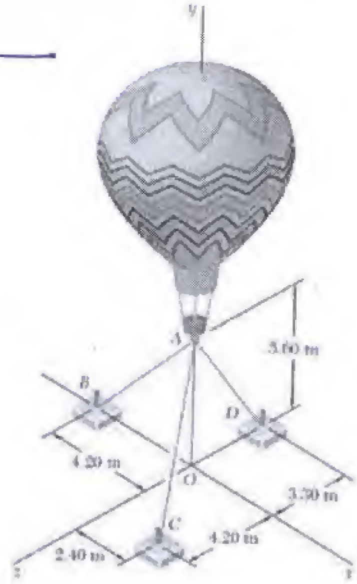
(1)

[1] ∴ the Co-ordinates of Point $A = (0, 5.6, 0)$

〃 〃 〃 〃 $B = (-4.2, 0, 0)$

〃 〃 〃 〃 $C = (2.4, 0, 4.2)$

〃 〃 〃 〃 $D = (0, 0, -3.3)$



∴ the force $\vec{P} = P\vec{J}$

∴ the force @ Cable $\overline{AB}$ is $\vec{T}_{AB} = T_{AB} * \hat{e}_{AB}$

$$\therefore \vec{T}_{AB} = T_{AB} * \frac{\overrightarrow{AB}}{|AB|} = T_{AB} * \frac{-4.2\vec{i} - 5.6\vec{J}}{\sqrt{(4.2)^2 + (5.6)^2}} = \frac{T_{AB}}{7}(-4.2\vec{i} - 5.6\vec{J})$$

∴ the force @ Cable $\overline{AC}$ is $\vec{T}_{AC} = T_{AC} * \hat{e}_{AC}$

$$\therefore \vec{T}_{AC} = T_{AC} * \frac{\overrightarrow{AC}}{|AC|} = T_{AC} * \frac{2.4\vec{i} - 5.6\vec{J} + 4.2\vec{k}}{\sqrt{(2.4)^2 + (5.6)^2 + (4.2)^2}} = \frac{T_{AC}}{7.4}(2.4\vec{i} - 5.6\vec{J} + 4.2\vec{k})$$

∴ The force @ Cable $\overline{AD}$ is $\vec{T}_{AD} = T_{AD} * \hat{e}_{AD}$

$$\therefore \vec{T}_{AD} = T_{AD} * \frac{\overrightarrow{AD}}{|AD|} = T_{AD} * \frac{-5.6\vec{J} - 3.3\vec{k}}{\sqrt{(5.6)^2 + (3.3)^2}} = \frac{T_{AD}}{6.5}(-5.6\vec{J} - 3.3\vec{k})$$

And ∴ the point (A) is in equilibrium state then,

$$\Sigma f_x = \text{zero} \; ; \quad -\frac{4.2}{7} T_{AB} + \frac{2.4}{7.4} T_{AC} = 0 \quad \& \; \therefore \underline{\underline{T_{AB} = 259 \, N}} \text{ "Given"}$$

$$\boxed{\therefore T_{AC} = 479.15 \, N}$$

$$\Sigma f_y = \text{zero} \; ; \quad P - \frac{5.6}{7} T_{AB} - \frac{5.6}{7.4} T_{AC} - \frac{5.6}{6.5} T_{AD} = 0 \longrightarrow \text{(I)}$$

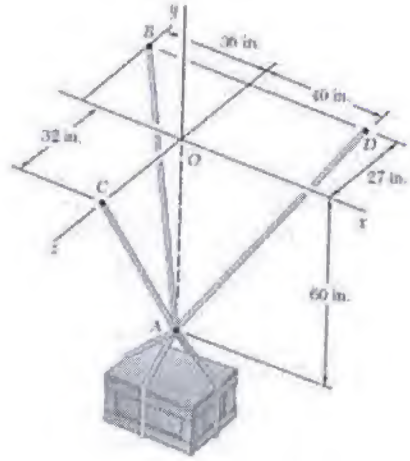
$$\Sigma f_z = \text{zero} \; ; \quad \frac{4.2}{7.4} T_{AC} - \frac{3.3}{6.5} T_{AD} = 0$$

$$\boxed{\therefore T_{AD} = 535.66 \, N}$$

∴ from Equ. (I) $\boxed{\therefore P = 1031.29 \, N}$

[2] Given: Each Cable Can Sustain a max. tension $= 616\ \text{lb}$.

Req.: The maximum weight of the Crate.



Solution:

The Co-ordinates of Point $A = (0, -60, 0)$

$B = (-36, 0, -27)$

$C = (0, 0, 32)$

$D = (40, 0, -27)$

In this problem we have 4-unknowns (the weight (ω) and Tension force in 3-Cables)

First: Get the direction of each force.

$$\vec{W} = -\omega \vec{J}$$

The force @ cable $\overline{AB}$ is $\vec{T}_{AB} = T_{AB} * \hat{e}_{AB}$

$$\vec{T}_{AB} = T_{AB} * \frac{-36\vec{i} + 60\vec{J} - 27\vec{k}}{\sqrt{(36)^2 + (60)^2 + (27)^2}} = \frac{T_{AB}}{75}(-36\vec{i} + 60\vec{J} - 27\vec{k})$$

The force @ Cable $\overline{AC}$ is $\vec{T}_{AC} = T_{AC} * \hat{e}_{AC}$

$$\vec{T}_{AC} = T_{AC} * \frac{0\vec{i} + 60\vec{J} + 32\vec{k}}{\sqrt{(60)^2 + (32)^2}} = \frac{T_{AC}}{68}(60\vec{J} + 32\vec{k})$$

The force @ Cable $\overline{AD}$ is $\vec{T}_{AD} = T_{AD} * \hat{e}_{AD}$

$$\vec{T}_{AD} = T_{AD} * \frac{40\vec{i} + 60\vec{J} - 27\vec{k}}{\sqrt{(40)^2 + (60)^2 + (27)^2}} = \frac{T_{AD}}{77}(40\vec{i} + 60\vec{J} - 27\vec{k})$$

Second: Put the equilibrium Equations.

$\sum f_x = \text{zero}$; $\quad -\frac{36}{75} T_{AB} + \frac{40}{77} T_{AD} = 0 \longrightarrow \text{I}$

$\sum f_y = \text{zero}$; $\quad -\omega + \frac{60}{75} T_{AB} + \frac{60}{68} T_{AC} + \frac{60}{77} T_{AD} = 0 \longrightarrow \text{II}$

$\sum f_z = \text{zero}$; $\quad -\frac{27}{75} T_{AB} + \frac{32}{68} T_{AC} - \frac{27}{77} T_{AD} = 0 \longrightarrow \text{III}$

So, we get 3-equations with 4-unknowns.

∴ we must assume one unknown to obtain others

∴ Assume that the tension at cable $\overline{AC}$ ⟹ "$T_{AC} = 616\text{-Ib}$"

from equ. (III) $\frac{27}{75} T_{AB} + \frac{27}{77} T_{AD} = \frac{32}{68} * 616 = 289.882 \rightarrow$ (III')

& equ. (I) $-\frac{36}{75} T_{AB} + \frac{40}{77} T_{AD} = zero \rightarrow$ (I)

∴ $\frac{2052}{77} T_{AD} = 289.882 * 36 \Longrightarrow \boxed{\therefore T_{AD} = 391.595\text{-lb}}$

$T_{AD} < 616\text{-Ib}$ ok.

∴ from equ. (I) $\boxed{T_{AB} = 423.8\text{-lb}}$

$T_{AB} < 616\text{-Ib}$ ok.

∴ from equ. (II) we can get the max. weight of the crate

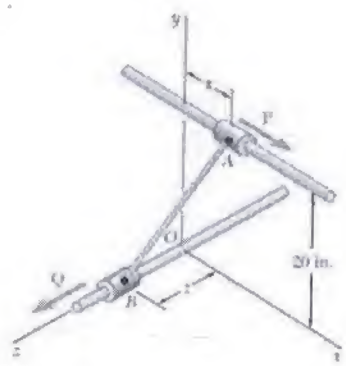
$$\therefore W = \frac{60}{75} * 423.8 + \frac{60}{68} * 616 + \frac{60}{77} * 391.595$$

$$\boxed{\therefore W = 1187.71\text{-Ib}}$$

3 Given: wire length = 25-in

force $Q = 60$-lb

$x = 9$-in



Req.: (a) Tension in wire

(b) the force (P)

Solution:

∴ the Co-ordinates of Point $A = (9, 20, 0)$

∴ the Co-ordinates of point $B = (0, 0, Z)$

$$\therefore \vec{BA} = A - B = 9\vec{i} + 20\vec{j} - Z\vec{k}$$

∴ the magnitude of the vector $|\vec{BA}|$ = the length of wire

$$\therefore |\vec{BA}| = \sqrt{(9)^2 + (20)^2 + (Z)^2} = 25 \Longrightarrow \therefore \underline{\underline{Z = 12\text{-in}}}$$

$$\therefore \vec{BA} = 9\vec{i} + 20\vec{j} - 12\vec{k}$$

∴ The point (B) is in Equilibrium State

∴ the tension in wire $\vec{T} = T * \hat{e}_{BA} = \frac{T}{25} * (9\vec{i} + 20\vec{j} - 12\vec{k})$

$$\therefore \vec{Q} = 60\vec{k} = \left(\frac{T}{25} * 12\right)\vec{k}$$

$$\boxed{\therefore T = 125\text{-lb}}$$

∴ The point (A) is in Equilibrium State

$$\therefore \vec{P} = P\vec{i} = \left(\frac{T}{25} * 9\right)\vec{i}$$

$$\boxed{\therefore P = 45\text{-lb}}$$

4] Given : Tension in Cable $\overline{BH} = 450\,N$

Req. : Moment about line $\overline{AD}$

Solution :

∴ The moment about line equal the moment about point in the line multiplied by the unit vector of the line.

$$\therefore M_{\overline{AD}} = \vec{M}_A \cdot \hat{e}_{AD}$$

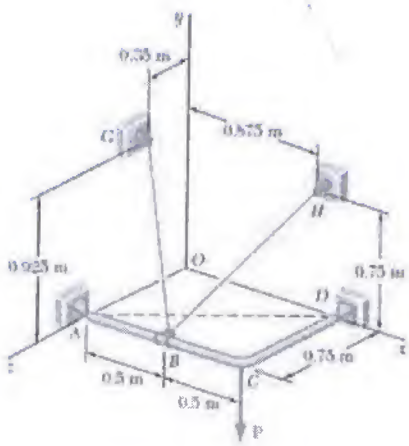
∴ the moment $\vec{M}_A = \vec{AB} \times \vec{F}_{BH}$

∴ The Co-ordinates of Point $A = (0, 0, 0.75)$

" " " " $B = (0.5, 0, 0.75)$

" " " " $D = (1.0, 0, 0)$

" " " " $H = (0.875, 0.75, 0)$



$$\therefore \vec{AB} = B - A = 0.5\,\vec{i}$$

$$\therefore \vec{F}_{BH} = T * \hat{e}_{BH} = 450 * \frac{0.375\,\vec{i} + 0.75\,\vec{j} - 0.75\,\vec{k}}{\sqrt{(0.375)^2 + (0.75)^2 * 2}} = 400 * (0.375, 0.75, -0.75)$$

$$\therefore \vec{F}_{BH} = 150\,\vec{i} + 300\,\vec{j} - 300\,\vec{k}$$

$$\therefore \vec{M}_A = \begin{vmatrix} i & j & k \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix} = \vec{i}\,(0) + \vec{j}\,(150) + \vec{k}\,(150)$$

$$\therefore \hat{e}_{AD} = \frac{\vec{AD}}{|AD|} = \frac{1.0\,\vec{i} - 0.75\,\vec{k}}{\sqrt{(1)^2 + (0.75)^2}} = 0.8\,\vec{i} - 0.6\,\vec{k}$$

$$\therefore M_{\overline{AD}} = \vec{M}_A \cdot \hat{e}_{AD} = (150\,\vec{j} + 150\,\vec{k}) \cdot (0.8\,\vec{i} - 0.6\,\vec{k}) = -150 * 0.6$$

$$\boxed{\therefore M_{\overline{AD}} = -90\ N.m}$$